Independent and conditionally independent counterfactuals

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CRoNoS & MDA 2019
Limassol
April 15, 2019

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Counterfactuals

The goal of the counterfactual analysis is the comparison between what actually happened to what would have happened under an alternative scenario.

How to define alternative scenarios?

- Exogenous policy change (Rothe, 2010),
- treatment group (Chernozhukov et al., 2013),
- filter the dependence between variables (this paper).

Quick literature overview

The vast majority of impact evaluation studies focus on parametric models

- treatment effect models (Heckman, 1978),
- propensity score matching (Rosenbaum and Rubin, 1983),
- matching estimators models (Abadie and Imbens, 2002),
- OLS, diff-in-diff estimators (Gertler et al., 2010).

Non-parametric methods

- propensity score through a nonparametric regression model (Heckman, et al. (1997, 1998)),
- non-parametric/parametric method (Chernozhukov et al., 2013)
  - under an assumption called conditional exogeneity counterfactual effects can be interpreted as causal effects.
- fully nonparametric approach
  - total effects (Rothe, 2010),
  - partial effects (Rothe, 2012).

This paper

Theoretical contribution

- provide a fully non-parametric dependence filtering framework
  - unconditional dependence,
  - conditional dependence,
- consistent inference methods
  - Gaussian and bootstrap confidence bounds,
- utilize smooth estimates (improved MSE performance),
- numerical verification.

Empirical contribution

- build a link to hypothesis testing,
- understand the dependence structure in the US grain market.

Unconditional setup (I)

General assumptions

- Y outcome variable (1d) with CDF given by $F_Y(y)$,
- X covariate (vector) with CDF/PDF given by $F_X(x)$ and $f_X(x)$,
- i.i.d sample $\{(Y_i, X_i) : i = 1, ..., n\}$.

Variable dependence

- $F_{Y|X}(y|x) \neq F_Y(y)$ for some x, y.

The dependence filtering idea

- counterfactual outcome variable Y' with realizations y',
- $F_{Y'|X}(y|x) = F_Y(y)$ for all x, y,
- estimate realizations y'.

Unconditional setup (II)

Filtering through data sharpening (Hall and Minote, 2002)

- assume $Y' = \phi(Y, X = x) \equiv \phi(Y)$,
- so that $y' = \phi(y, x) \equiv \phi(y)$
- $\phi : \mathbb{R}^{d_X+1} \to \mathbb{R}$ and localy invertible.

Then the plug-in estimator of the joint density becomes

$$\hat{F}_{Y'|X}(y, x) = \frac{n^{-1} \sum_{i=1}^{n} \bar{K}_{\mathbf{H_Y}} \left(y - \phi(Y_i)\right) K_{\mathbf{H_X}} \left(x - X_i\right)}{n^{-1} \sum_{i=1}^{n} K_{\mathbf{H_X}} \left(x - X_i\right)} = F_Y(y), \quad (1)$$

where $\mathbf{H}_{\mathbf{H_Y}}$ and $\mathbf{H}_{\mathbf{H_X}}$ are bandwidth matrices, $\bar{K}_{\mathbf{H_Y}}$ is a cumulative and $K_{\mathbf{H_X}}$ is a scaled (multivariate) kernel function satisfying the standard regularity conditions (Wand and Jones, 1995).

Unconditional setup (III)

Theorem

Suppose that we have an i.i.d. sample $\{(Y_i, \mathbf{X_i}) : i = 1, ..., n\}$ *from a continuous distribution with well-defined and sufficiently smooth PDFs. Then, the counterfactual random variable* Y' *with realizations* y'*, satisfying the independence condition given by* $F_{Y'|X}(y|x) = F_Y(y)$*, follows*

$$F_Y(y') = F_{Y|\mathbf{X}}(y|\mathbf{x}), \tag{2}$$

where $F_{Y|\mathbf{X}}$ *is the conditional distribution function of* Y *given* $\mathbf{X} = \mathbf{x}$*, for any* y *and* $\mathbf{x}$ *in the support of* $(Y, \mathbf{X})$*.*

Unconditional setup (estimation)

The estimator of the independent counterfactuals

$$\hat{y}' = \hat{F}_Y^{-1}(\hat{F}_{Y|\mathbf{x}}(y|\mathbf{x})). \tag{3}$$

Theorem

Suppose that Assumptions 1-4 hold (see Appendix). Then

$$\sqrt{n}\left(\hat{y}' - y'\right) \xrightarrow{d} N(0, \sigma^2), \tag{4}$$

conditional on the data, where σ^2 is given by

$$\sigma^2 = \frac{F_{Y|X}(y|x)(1 - F_{Y|X}(y|x))}{f_Y\left(F_Y^{-1}(F_{Y|X}(y|x))\right)} + \frac{\int K(u)^2 \mathrm{d}u}{f_X(x)\Pi_{j=1}^{d_X} h_{jXY}} \frac{F_{Y|X}(y|x)(1 - F_{Y|X}(y|x))}{f_Y\left(F_Y^{-1}(F_{Y|X}(y|x))\right)}. \tag{5}$$

Unconditional setup (consistency)

Consistency of $\hat{y}'$ achieved under uniform convergence of estimators

- satisfied for 1-dimensional X,
- higher dimensions require lower estimate bias (higher order kernels)

Assumption (Bandwidths of conditional CDF)

As $n \to \infty$,

1. $n^{1/2} h_Y / (\log n)^{1/2} + n^{1/2} h_Y^r \to 0,$
2. $n^{1/2} h_X / \log n + n^{1/2} h_X^r \to 0,$

where r is the kernel order.

Unconditional setup (example)

Consider a stylized mean-dependent process

$$\begin{aligned} X &\sim N(0,1), \\ y_t &= ax_t + \sqrt{1-a^2}\varepsilon_t, \quad a \in (0,1). \end{aligned} \tag{6}$$

Independent counterfactuals are equal to the error term

$$\begin{aligned} y'_t &\equiv \phi(y_t, x_t) = F_Y^{-1}(F_{Y|X}(y_t|x_t)) \\ &= \sqrt{2}\mathrm{erf}^{-1}\left(\mathrm{erf}\left(\frac{y_t - ax_t}{\sqrt{2-2a^2}}\right)\right) = \frac{y_t - ax_t}{\sqrt{1-a^2}} = \varepsilon_t. \end{aligned} \tag{7}$$

More generally, under error exogeneity for nonseparable model

$$\begin{aligned} y_t &= m(x_t, \varepsilon_t), \\ y'_t &= F_Y^{-1}(F_\varepsilon(\varepsilon_t)). \end{aligned} \tag{8}$$

Conditional setup(I)

General assumptions

- Y outcome variable with CDF given by $F_Y(y)$,
- Q variable(s) with CDF/PDF given by $F_Q(q)$ and $f_q(q)$,
- X covariate (vector) with CDF/PDF given by $F_X(x)$ and $f_X(x)$,
- i.i.d sample $\{(Y_i, Q_i, X_i) : i = 1, ..., n\}$.

Variable dependence (CDF version of Diks and Panchenko, 2006)

- $F_{Y|Q,X}(y|q,x) \neq F_{Y|Q}(y|q)$ for some y, q, x.

The filtering idea

- counterfactual outcome variable Y'' with realizations y'',
- $F_{Y''|Q,X}(y|q,x) = F_{Y|Q}(y|q)$ for all y, q, x,
- estimate realizations y''.

Conditional setup (II)

Theorem

Suppose that we have an i.i.d. sample $\{(Y_i, \mathbf{Q_i}, \mathbf{X_i}) : i = 1, ..., n\}$ *from a continuous distribution with well-defined and sufficiently smooth PDFs. Then, the counterfactual random variable* Y''*, satisfying the conditional independence condition given by* $F_{Y''|Q,X}(y|q,x) = F_{Y|Q}(y|q)$*, follows asymptotically*

$$F_{Y|\mathbf{Q}}(y''|\mathbf{q}) = F_{Y|\mathbf{Q},\mathbf{X}}(y|\mathbf{q},\mathbf{x}). \tag{9}$$

Monte Carlo setup

Process specification (Diks and Wolski (2016))

$$\begin{aligned} X_i &\sim \mathrm{N}(0,1), \\ Y_i &\sim \mathrm{N}\left(0, c + aX_i^2\right), \end{aligned} \tag{10}$$

with $c > 0$ and $1 > a > 0$.

Filtering Mean Squared Error (MSE) is given by

$$\mathrm{MSE}(\hat{y}') = n^{-1} \sum_{i=1}^{n} \left(\hat{F}_Y^{-1}(\hat{F}_{Y|\mathbf{X}}^{-i}(y|\mathbf{x})) - F_Y^{-1}(F_{Y|\mathbf{X}}(y|\mathbf{x})) \right)^2 .$$

Technicalities

- compare step-wise and smooth kernel estimators (normal-scale, process-driven, LS-CV bandwidths)
- 1000 replications.

MSE performance

Table: Median MSE estimates of independent counterfactuals.

Bandwidth selector	n=50	n=100	n=200	n=500	n=1000
no smoothing	0.584	0.406	0.274	0.169	0.107
smoothing	0.292	0.232	0.178	0.116	0.080

Notes: Medians taken over 1000 Monte Carlo results for the ARCH process. Bandwidth selectors are chosen as: 'no smoothing' for step-wise estimators and 'smoothing' for normal-scale bandwidth selector.

Granger causality in the US grain market

Null hypothesis

$$\{X_t\} \text{ is not Granger causing } \{Y_t\}.$$

Diks & Wolski (2016)

- framework

$$Y_{t+1}|(X_t, Y_t) \sim Y_{t+1}|Y_t,$$

- test statistic (implication of the null)

$$q \equiv E\left[f_{X,Y,Z}(X,Y,Z)f_Y(Y) - f_{X,Y}(X,Y)f_{Y,Z}(Y,Z)\right] = 0.$$

Conditionally independent counterfactuals

- framework

$$F_{Y''_{t+1}|Y_t,X_t}(y_{t+1}|y_t,x_t) = F_{Y_{t+1}|Y_t}(y_{t+1}|y_t)\ ,$$

- test statistic

$$z \equiv \frac{\hat{y}''_{t+1} - y_{t+1}}{\hat{\sigma}}.$$

Granger causality in the US grain market

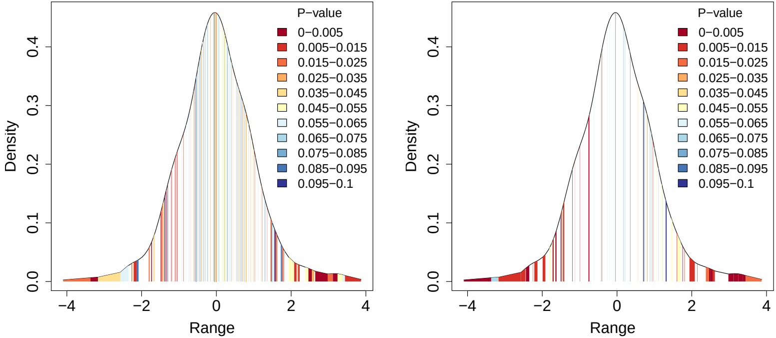
Table: US grain market results (Diks & Wolski, 2016).

Variables		Linear Granger Causality				Nonlinear Granger Causality (N)			
		Raw data		VAR residuals		Raw data		VAR residuals	
X	Y	X→Y	Y→X	X→Y	Y→X	X→Y	Y→X	X→Y	Y→X
Corn	Wheat					***	***	***	***
Corn	Beans		*						
Beans	Wheat								

Notes: causality results for the pairwise relations of the log returns on the US grain market. (*), (**), (***) denote statistical significance at 10%, 5% and 1%. Period: 09/01/2010–03/06/2013. Nonlinear tests are performed on standardized data, transformed to (N)ormal marginals. The number of lags is $l_X = l_Y = 1$ *from the Bayesian Information Criterion.*

Granger causality in the US grain market

Figure: Wheat counterfactuals independent from corn (left) and beans (right).



Notes: causality results for the pairwise relations of the log returns on the corn → wheat (left panel) and beans → wheat (right panel). Period: 09/01/2010–03/06/2013. Nonlinear tests are performed on standardized data, transformed to (N)ormal marginals. The number of lags is $l_X = l_Y = 1$ *from the Bayesian Information Criterion.*

The main take-aways

Theory

- fully non-parametric dependence filtering framework,
  - unconditional + conditional dependence,
- standard + bootstrap confidence bounds,
- desired MSE performance on non-linear processes,
- good finite-sample properties.

Practice

- framework flexibility,
- hypothesis testing
  - further insights into the US grain market dependence structure.

In the future

- panel data extension,
- causal interpretation (yes, under error exogeneity).

Selected references

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The End

Estimation assumptions (1)

Assumption (1)

Data $\{\mathbf{W}_i : i = 1, ..., n\}$, *where* $\mathbf{W}_i = \{W_{1i}, ..., W_{d_W i}\}$, *are i.i.d. as a* d_W*-variate smooth continuous distribution* $F_{\mathbf{W}}(\mathbf{w})$ *with well-defined PDF* $f_{\mathbf{W}}(\mathbf{w})$ *and respective derivatives, up to a finite order* r*, which are finite, continuous and uniformly bounded on the support.*

Estimation assumptions (2)

Assumption (2)

Kernel function $K : \mathbb{R}^{d_W} \to \mathbb{R}$ *behaves as*

$$\int K(\mathbf{w})\mathrm{d}\mathbf{w} = 1,$$
$$\int K(\mathbf{w})\mathbf{w}^c\mathrm{d}\mathbf{w} = 0 \quad \text{for} \quad c = 1, ..., r-1,$$
$$\int K(\mathbf{w})\mathbf{w}^c\mathrm{d}\mathbf{w} = \kappa_r I_{d_W} < \infty \quad \text{for} \quad c = r, \tag{11}$$

and $K(\mathbf{w})$ *is* r*-times differentiable, where* I_{d_W} *is a* $d_W \times d_W$ *identity matrix.*

Estimation assumptions (3/4)

Assumption (3)

As $n \to \infty$,

1. $n^{1/2} h_0 / (\log n)^{1/2} + n^{1/2} h_0^r \to 0$,
2. $n^{1/2} \det \mathbf{H}^{1/2} / \log n + n^{1/2} \max \mathbf{H}^{r/2} \to 0$.

Assumption (4)

We assume that (i) distribution functions F_Y *and* $F_{Y|\mathbf{X}}$ *are Hadamard differentiable, (ii)* F_Y^{-1} *is uniformly Lipschitz and bounded by* $[a, b] \in \mathbb{R}$*, (iii)* Y *is supported by a compact interval on* $J \in \mathbb{R}$ *for which* $F_{Y|\mathbf{X}}(y|\mathbf{x})$ *is uniformly bounded by* $[p_1, p_2] \in (0, 1)$.